Signature-based validation of real-world economic scenarios

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Résumé

We propose a new approach for the validation of real-world economic scenarios motivated by insurance applications. This approach is based on the statistical test developed by Chevyrev and Oberhauser (2022) and relies on the notions of signature and maximum mean distance. This test allows to check whether two samples of stochastic processes paths come from the same distribution. Our contribution is to apply this test to a variety of one-dimensional stochastic processes relevant for the modelling of equity stock price and volatility as well as inflation in view of actuarial applications. At first, we present a numerical analysis with synthetic data in order to measure the statistical power of the test and then, we work with historical data to study the ability of the test to discriminate between several models in practice. These numerical experiments are conducted under two constraints: 1. we consider an asymmetric setting in which we compare a large sample of simulated real-world scenarios and a small sample that consists of (or represents in the synthetic data case) historical data, both with a monthly time step as often considered in practice and 2. we make the two samples identical from the perspective of validation methods used in practice, i.e. we impose that the marginal distributions of the two samples are the same or very close at a given one-year horizon. By performing specific transformations of the signature, we obtain statistical powers close to 1 in this framework. Moreover, we show that some models are rejected and others are not when applying the test against historical data. These numerical results demonstrate the potential of this validation approach for real-world economic scenarios and more generally for any application requiring to exhibit the consistency of a stochastic model with historical paths. We also discuss several challenges related to the numerical implementation of this approach, and highlight its domain of validity in terms of the distance between models and the volume of data at hand.

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